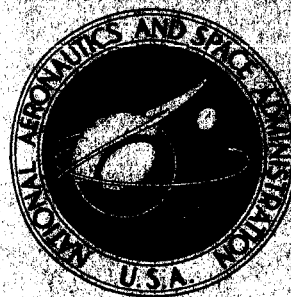


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CYCLOTRON WAVES IN A
TWO ION SPECIES PLASMA**

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SUMMARY

Calculations were made on the efficiency of using a Stix coil to transfer power to waves near the ion cyclotron frequencies of a cold, collisionless plasma consisting of two ion species. The mass ratio m_b/m_a of the ion species was 2, and the frequency region near and between the cyclotron frequencies (Ω_{ia} and Ω_{ib}) of each ion species was investigated. For wave frequencies ω less than Ω_{ib} , only the usual ion cyclotron resonance absorption peak is obtained. For $\Omega_{ia} > \omega > \Omega_{ib}$, two absorption peaks are obtained. The additional peak corresponds to power absorbed by fast wave modes associated with the heavier species. The wave set up nearer the cyclotron frequency of the lighter ion Ω_{ia} is a left-polarized wave, and power is absorbed primarily by modes whose wavelengths, which are parallel to the static magnetic field, are near the wavelength of the Stix coil. The second absorption peak occurs at lower frequencies where the power is absorbed by the right-polarized fast wave modes associated with the heavier species. There is a sharp cutoff on the low frequency side of this second absorption peak at

$$\omega = \left[\Omega_{ia} \Omega_{ib} \frac{x \Omega_{ia} + \Omega_{ib}}{x \Omega_{ib} + \Omega_{ia}} \right]^{1/2}$$

where x is the density ratio of the two species, n_b/n_a . When x is made small the latter peak disappears.

INTRODUCTION

Previously, extensive calculations were made (ref. 1) on the efficiency of transferring power from a Stix coil to waves in a cold, collision-free hydrogen plasma at frequencies near the ion cyclotron frequency. In these calculations, the plasma was assumed to contain only electrons and a single ion species, and the effects of electron density, inhomogeneities, Stix coil wavelength, etc., on power transfer were investigated.

However, plasmas used for the study of ion cyclotron waves often contain more than a single ion species, such as H^+ , H_2^+ , D^+ , and D_2^+ . Extending the calculations of reference 1 to include the effects of a second ion species therefore has practical as well as theoretical interest. Since the mass ratios $H_2^+:H^+$, $D^+:H^+$, and $D_2^+:D^+$ are all 2 and since it would be unusual to have a plasma consisting primarily of D_2^+ and H^+ ions, a mass ratio of 2 was chosen for these calculations. In this report calculations are presented to show how power transfer varies with magnetic field in the region near and between the cyclotron frequencies of the two ion species as the relative concentration of the two species is varied.

THEORY

The basic equations to be solved are the same regardless of the number of species present in the plasma. These equations are given in references 1 and 2 where Maxwell's equations and the particle equations of motion (in polar or rectangular coordinates) are combined and written for a cold, collisionless plasma as

$$\underline{\nabla} \times \underline{\nabla} \times \underline{E} = \frac{\omega^2}{c^2} \underline{K} \cdot \underline{E} \quad (1)$$

where it has been assumed that all field quantities are of the form

$$F(\underline{r}, t) = f(r)e^{i(kz - \omega t)} \quad (2)$$

and

$$\underline{K} = \begin{pmatrix} S & -iD & 0 \\ iD & S & 0 \\ 0 & 0 & P \end{pmatrix} \quad (3)$$

The terms S , P , and D in the dielectric tensor \underline{K} are given in chapter 1 of reference 2. They depend on the wave frequency ω , the plasma frequencies

$$\Pi_k^2 = \frac{n_k Z_k e^2}{m_k} \quad (k = e, a, b) \quad (4)$$

of the various species, and the cyclotron frequencies

$$\Omega_k = \left| \frac{z_k e B_0}{m_k} \right| \quad (k = e, a, b) \quad (5)$$

where B_0 is the static magnetic field in the z -direction.

The term which is important in computing power transfer to the plasma is the azimuthal component of the electric field E_θ , which is written as

$$E_\theta = \left[a_1 J_1(\nu_1 r) + a_2 J_1(\nu_2 r) \right] e^{i(kz - \omega t)} \quad (6)$$

where k is the wave number in the direction of the static magnetic field. The coefficients a_1 and a_2 are determined by the boundary conditions.

The term P is determined primarily by the electrons, so that the terms S and D are the ones most affected by the presence of a second ion species. The variations of ν_1 and ν_2 with frequency are determined mostly by the properties of S and D . The terms ν_1 and ν_2 are given by

$$\left. \begin{aligned} \nu_1^2 &= \frac{-B + \sqrt{B^2 - 4C}}{2} \\ \nu_2^2 &= \frac{-B - \sqrt{B^2 - 4C}}{2} \end{aligned} \right\} \quad (7)$$

$$B = \frac{k^2(P + S)}{S} - \frac{\omega^2}{c^2} \left(\frac{PS + S^2 - D^2}{S} \right)$$

$$C = \frac{P}{S} \left[k^4 - 2 \frac{\omega^2}{c^2} k^2 S + \frac{\omega^4}{c^4} (S^2 - D^2) \right]$$

Figure 1 contains plots of ν_1^2 and ν_2^2 against k^2 for several frequency regions. In some of these plots, there are two branches for which k^2 is positive. A branch which intercepts the k^2 axis at points marked R has right polarization, while those having intercepts at points marked L are left polarized. The species a and b are labeled so that $m_b/m_a = 2$. In this report, only those plasmas are considered whose electron density is so low (usually below 10^{14} cm^{-3}) that the right polarized wave is not able to propagate in a finite radius plasma column when only a single ion species is present. This right polarized wave has $k^2 < (\omega^2/c^2)R$.

First, consider the region in the frequency spectrum (fig. 1) to the right of $\omega = \Omega_{ib}$ where Ω_{ib} is the cyclotron frequency of the heavier mass ion species. The calculations

show that here the presence of species a only changes the power transfer characteristics from a Stix coil to the ion cyclotron wave quantitatively and not to a large enough degree to elaborate on in this report.

Because the density is low, there can be no wave propagation at all for frequencies to the left of $\omega = \Omega_{ia}$, where Ω_{ia} is the cyclotron frequency of the lighter species.

Between the frequencies Ω_{ia} and Ω_{ib} (fig. 1) there are two special points marked $D = 0$ and $S = 0$. The terms S and D can be zero in the frequency range $\Omega_{ia} > \omega > \Omega_{ib}$ only when more than one ion species is present; it is the way S and D vary with frequency and/or magnetic field that leads to some interesting power transfer results. The conditions for $S = 0$ are

$$\omega^2 = \omega_S^2 \approx \Omega_{ia}\Omega_{ib} \left(\frac{X_a \Omega_{ib} + X_b \Omega_{ia}}{X_a \Omega_{ia} + X_b \Omega_{ib}} \right) \quad (8a)$$

and for $D = 0$

$$\omega^2 = \omega_D^2 = X_b \Omega_{ia}^2 + X_a \Omega_{ib}^2 \quad (8b)$$

where X_a and X_b are the relative concentrations of the species ($X_a + X_b = 1$). Waves which can be set up by the Stix coil in the region to the left of $D = 0$ ($\omega > \omega_D$) are left polarized and those to the right are right polarized. From equation (1), it can be shown that $E_\theta/E_r = iD/(n^2 - S)$, and as has been pointed out by another investigator (private communication with M. A. Rothman of Princeton University) $E_\theta = 0$ at $D = 0$ so that the Stix coil should be ineffective in coupling power to plasma waves at this point.

RESULTS AND DISCUSSION

Calculations of power transfer were made from a Stix coil to waves in the plasma for $\omega > \Omega_{ia}$ with the relative concentrations of the two ion species the principal variable. For these calculations, the Stix coil wavelength was 45 centimeters, the coil radius 10 centimeters, the plasma radius 5 centimeters, and electron as well as the total ion density was 10^{13} per cubic centimeter. The results of these calculations are shown in figure 2 where relative power transfer P^* (defined in ref. 1) is plotted against $\Omega = \omega/\Omega_{ia}$ for several X_b/X_a ratios. The term P^* is the total power transferred to the plasma wave from the Stix coil divided by $2\pi a^2 j^2$

$$p^* = \frac{\text{Total power transfer}}{2\pi a^2 j^{*2}} \quad (9)$$

where j^* is the amplitude of the current density in the current sheet (i.e., Stix coil) which extends from $-a \leq z \leq a$.

As the relative concentration of the heavier species is increased, the following features can be seen in figure 2:

- (1) The amplitude of the resonance peak closest to $\omega = \Omega_{ia}$ decreases.
- (2) The resonance peak moves closer to $\omega = \Omega_{ia}$ and becomes narrower.
- (3) A second peak appears to the low frequency side of the first resonance peak.
- (4) The amplitude of the second peak increases and the peak becomes narrower.
- (5) The resonance peak of the heavier species near $\omega = \Omega_{ib}$ moves to the right and broadens, as expected (ref. 1).

Calculations show that for the resonance peak nearest Ω_{ia} , power is absorbed primarily by modes whose wavelengths, which are parallel to the static magnetic field, are near the wavelength of the Stix coil and that the wave is left polarized. This resonance peak is the normal ion cyclotron resonance peak associated with the lighter species. To show how the heavier species affects the position of this resonance peak, the arrows in figure 2 indicate the position of the peak if the heavier species were absent and the ion and electron density were equal to the ion density of the lighter species in the mixture. For the absorption peak for which $\omega_D > \omega > \omega_S$, the wavelength of the modes absorbing the power is considerably longer than the Stix coil wavelength, and the waves are right polarized.

In addition, it should be noticed that there exists a sharp cutoff on the low frequency side of the peak just to the left of $\omega = \omega_S$, and, for the conditions of the model, no power is transferred to the plasma between this cutoff point and $\omega = \Omega_{ib}$. Also, the amplitude of the resonance peak near $\omega = \Omega_{ib}$ is not severely reduced until the ion density of the heavier species becomes less than about 10^{12} per cubic centimeter (ref. 1).

Thus, the presence of the lighter species does not severely affect power transfer near the cyclotron frequency of the heavier species, but the presence of the heavier species can severely affect power transfer near the cyclotron frequency of the lighter species.

An examination of the curves for ν^2 against k^2 (fig. 1) reveals that the modes absorbing power in the peak nearest $\omega = \omega_S$ are the fast wave modes (i.e., waves with lower k values) associated with the heavier species. When the lighter species is absent, these modes absorb power at different frequencies, and there are distinct absorption peaks for each of the modes. When a lighter species is introduced, all the fast wave modes associated with the heavier species are squeezed together, and all absorb power near $\omega = \omega_S$. In addition, there are fast wave modes associated with the

lighter species which absorb power for $\omega > \Omega_{ia}$. The absorption peaks for these modes are not shown in figure 2.

To examine the effect of increasing the electron density, a power transfer calculation was made in the region $\Omega_{ia} > \omega > \Omega_{ib}$ for $n_e = 10^{14}$ per cubic centimeter, $X_a = 0.4$, $X_b = 0.6$, and all other conditions the same as those for figure 2. This power transfer curve is given in figure 3, where it can be seen that, at $n_e = 10^{14}$ per cubic centimeter, the ion cyclotron peak has virtually disappeared, while the peak near $\omega = \omega_S$ is much larger. This occurs because the fast wave modes associated with the heavier species now have wavelengths close to the wavelength of the Stix coil when ω is near ω_S . For $n_e < 10^{12}$ per cubic centimeter, there is no peak near $\omega = \omega_S$, since the fast wave modes do not absorb power efficiently at low densities.

CONCLUSION

When a second ion species is included in the calculations of power transfer to plasma waves in the vicinity of either ion cyclotron frequency, additional features are found in the absorption spectrum. As the relative concentration of the heavier species is increased, a peak, which corresponds to power absorption by right polarized waves associated with the heavier ion species, appears near ω_S (wave frequency at $S = 0$). In addition, the normal left polarized cyclotron peaks are changed in magnitude and position. For high densities ($n_e > 10^{14}$), this peak, corresponding to right polarized waves, dominates the spectrum.

Lewis Research Center

National Aeronautics and Space Administration

Cleveland, Ohio, August 31, 1967,

129-01-05-09-22.

APPENDIX - SYMBOLS

a	one-half of Stix coil length	r	radial cylindrical coordinate
B	$k^2 \frac{(P + S)}{S} - \frac{\omega^2}{c^2} \left(\frac{PS + S^2 - D^2}{S} \right)$	S	defined in ref. 1
B ₀	static magnetic field parallel to z-axis	t	time
C	$\frac{P}{S} \left[k^4 - 2 \frac{\omega^2}{c^2} k^2 S + \frac{\omega^4}{c^4} (S^2 - D^2) \right]$	X	ratio of heavier to lighter ion species concentration
c	speed of light	x	relative ion species concentration
D	defined in ref. 1	z	axial cylindrical coordinate
<u>E</u>	electric field vector (time varying)	Z _k	charge number for particles of type k
e	electronic charge	θ	azimuthal cylindrical coordinate
F(<u>r</u> , t)	electric or magnetic field quantity	ν ₁ , ν ₂	defined by eq. (7)
f(r)	radial variation of F(<u>r</u> , t)	Π	plasma frequency
j*	current density of wave exciting current sheet	Ω	cyclotron frequency (or frequency ratio)
k	wave number	ω	wave frequency
<u>K</u>	dielectric tensor	ω _S , ω _D	ω when S or D equals zero
L	defined in ref. 1	Subscripts:	
m	species mass	a	lighter ion species
n	density	b	heavier ion species
P	defined in ref. 1	e	electron
P*	relative power transfer to plasma	i	ion
R	defined in ref. 1	k	particle species
		r	radial coordinate
		z	axial coordinate
		θ	azimuthal coordinate

REFERENCES

1. Sigman, Donald R. ; and Reinmann, John J. : Ion Cyclotron Wave Generation in Uniform and Nonuniform Plasma Including Electron Inertia Effects. NASA TN D-4058, 1967.
2. Stix, Thomas H. : The Theory of Plasma Waves. McGraw-Hill Book Co., Inc., 1962.

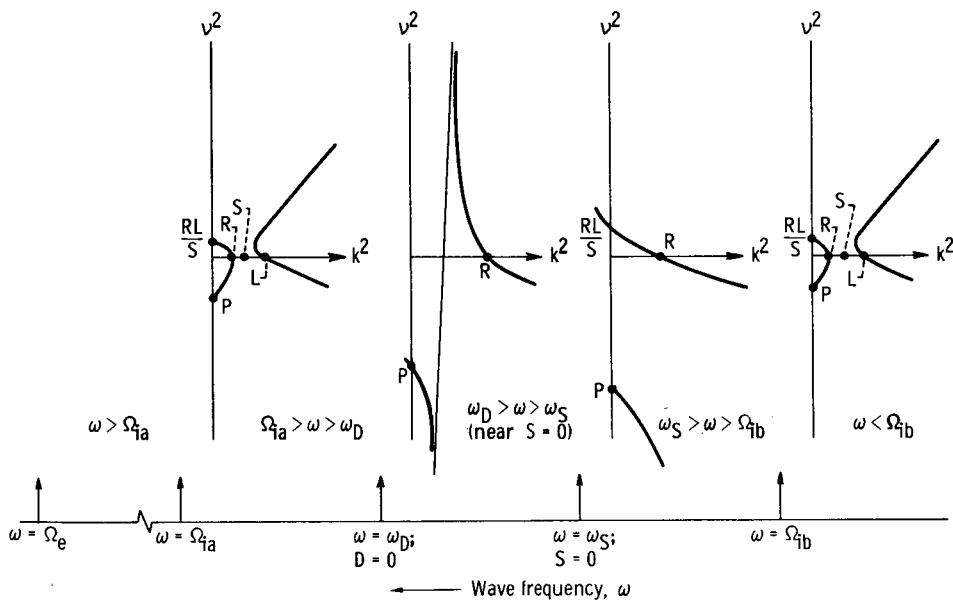
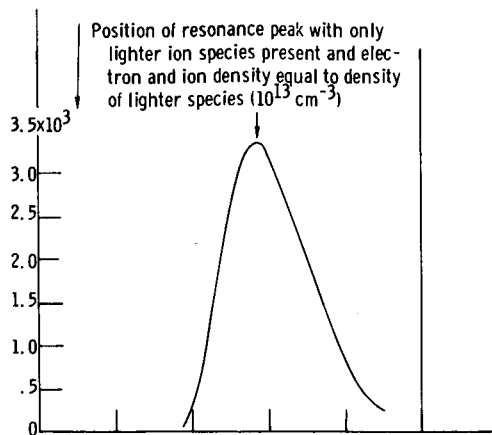
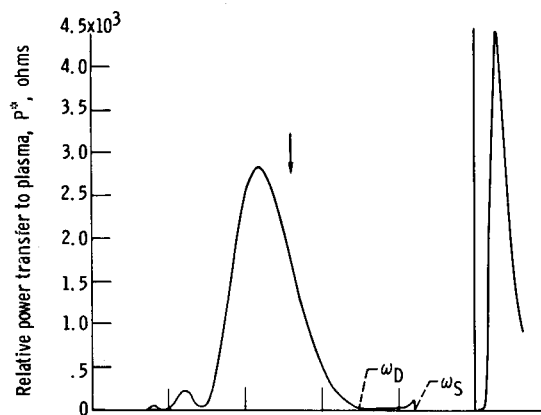


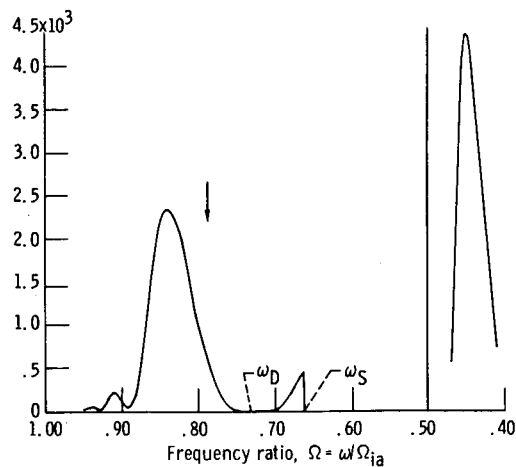
Figure 1. - Dispersion relation for two ion species plasma in various frequency ranges.



(a) Concentration ratio, $X_a:X_b$, 1:0.



(b) Concentration ratio, $X_a:X_b$, 0.8:0.2.



(c) Concentration ratio, $X_a:X_b$, 0.6:0.4.

Figure 2. - Relative power transfer as function of frequency ratio ($\Omega = \omega/\Omega_{ia}$). Stix coil wavelength, 45 centimeters; coil radius, 10 centimeters; plasma radius, 5 centimeters; electron number density, 10^{13} per cubic centimeter.

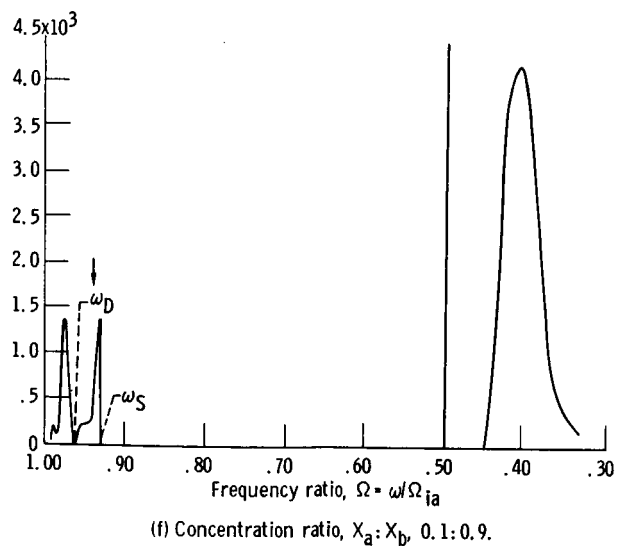
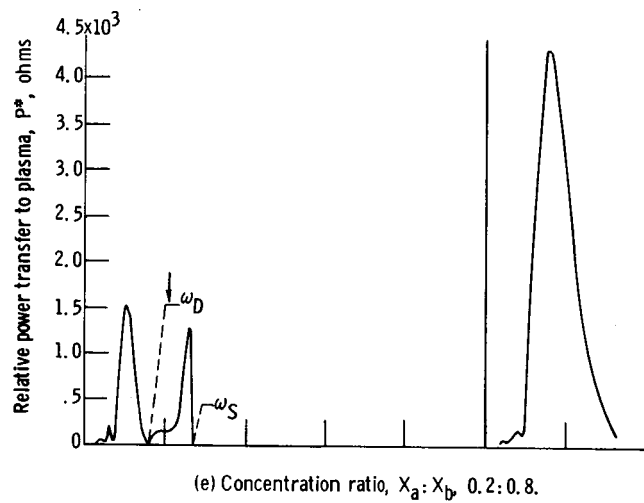
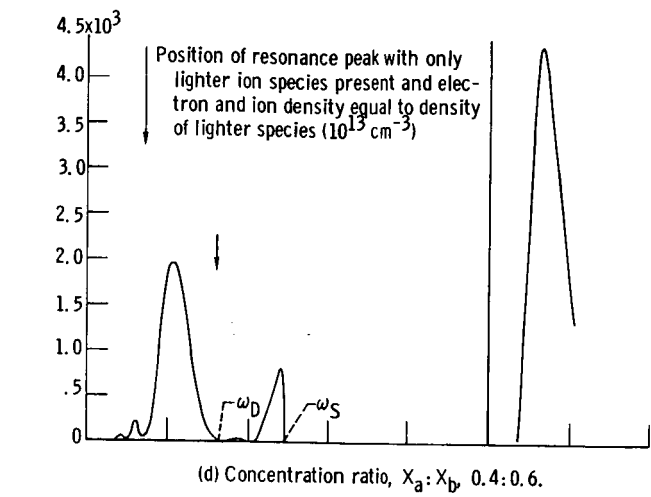


Figure 2. - Concluded.

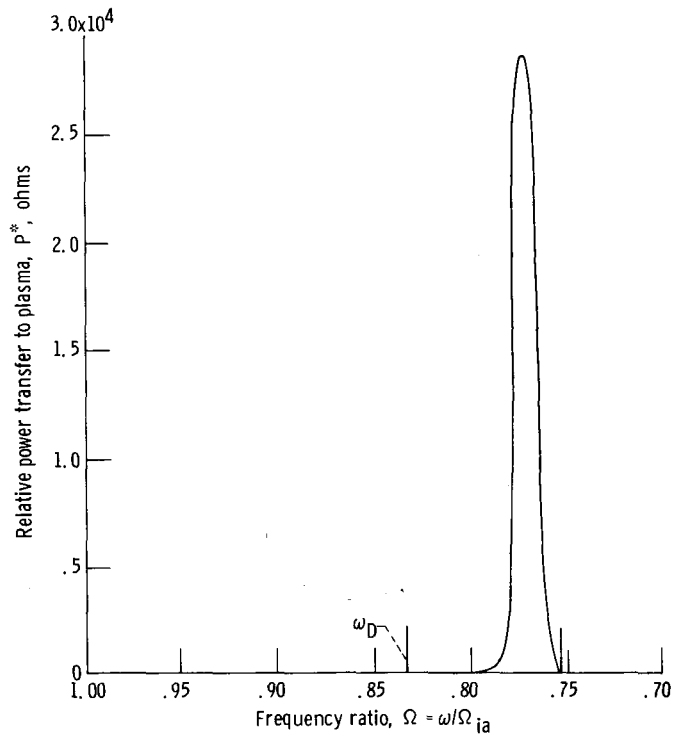


Figure 3. - Relative power transfer as function of frequency ratio for electron number density of 10^{14} per cubic centimeter. Stix coil wavelength, 45 centimeters; coil radius, 10 centimeters; plasma radius, 5 centimeters; concentration ratio, $X_a:X_b$, 0.4:0.6.